## Choice and Chance

All human actions have one or more of these seven causes: chance, nature, compulsions, habit, reason, passion and desire.

Aristotle 384 BC-322 BC

## Note

This presentation is available in pdf format at rhoadley.org/presentations

Further information is available at rhoadley.org

Most of the audio and video examples are available separately on the WebCT implementation of this presentation:
http://webct.anglia.ac.uk/

Choice and Chance Natural Forms

Hills and Water from Applecross, Scotland, April 2007


Use the lines of the hills in the background


- Then increase the rate until the drips start to come together into a stream.
- This is the point at which you have chaotic behaviour as the tap 'decides' whether it's output should be a drip or a stream.



## Choice and Chance

## Natural Forms American Beauty



## Excerpt from the film American Beauty (1999)

Notice the music here. Does this reflect the 'randomness' of the plastic bag at all?



Natural Forms Clouds

Natural Forms Clouds


Natural Forms Flocks


Flock

## Natural Forms Pebbles

Random, and yet patterned. Because we don't expect an explicit pattern, we're not concerned if it's not there. Indeed, any explicit patterning would be seen as 'unnatural'.


Natural Forms Leaves
As pebbles...


- Probability theory
- Information Theory
- Entropy
- Algorithmic Randomness
- Algorithmic Probability
- Chaos Theory
- Cryptography
- Game Theory
- Pattern Recognition
- Hidden Variable Theories
- Communication Theory
- Noise

Natural and Mathematical Forms Images


Natural and Mathematical Forms Fractal Broccoli


Natural and Mathematical Forms Fractal Fruit


Natural and Mathematical Forms Fractal Garden



- Popular perceptions of randomness are frequently wrong, based on logical fallacies. The following is an attempt to identify the source of such fallacies and correct the logical errors.


## A number is "due"

- This argument says that "since all numbers will eventually appear in a random selection, those that have not come up yet are 'due' and thus more likely to come up soon". This logic is only correct if applied to a system where numbers that come up are removed from the system, such as when playing cards are drawn and not returned to the deck.
- Once a jack is removed from the deck, the next draw is less likely to be a jack and more likely to be some other card. However, if the jack is returned to the deck, and the deck is thoroughly reshuffled, there is an equal chance of drawing a jack or any other card the next time.
- The same truth applies to any other case where objects are selected independently and nothing is removed from the system after each event, such as a die roll, coin toss or most lottery number selection schemes.
- A way to look at it is to note that random processes such as throwing coins don't have memory, making it impossible for past outcomes to affect the present and


## A number is "cursed"

This argument is almost the reverse of the above, and says that numbers which have come up less often in the past will continue to come up less often in the future. A similar "number is 'blessed'" argument might be made saying that numbers which have come up more often in the past are likely to do so in the future. This logic is only valid if the roll is somehow biased and results don't have equal probabilities - for example, with weighted dice. If we know for certain that the roll is fair, then previous events have no influence over future events.

Note that in nature, unexpected or uncertain events rarely occur with perfectly equal frequencies so learning which events are likely to have higher probability by observing outcomes makes sense. What is fallacious is to apply this logic to systems which are specially designed so that all outcomes are equally likely - such as dice, roulette wheels, and so on.

- Randomness is an objective property.
- What appears random to one observer may not appear random to another observer.
- Consider two observers of a sequence of bits, only one of which who has the cryptographic key needed to turn the sequence of bits into a readable message. The message is not random, but is for one of the observers unpredictable.
- One of the intriguing aspects of random processes is that it is hard to know whether the process is truly random. The observer can always suspect that there is some "key" that unlocks the message. This is one of the foundations of superstition and is also what is a driving motive, curiosity, for discovery in science and mathematics.
- Under the cosmological hypothesis of determinism there is no randomness in the universe, only unpredictability.

```
    M, l115011,0434425ee235534323355003213
    2423454,01231021355550544212344133213
    4035423.0204433225554424e32054e222e213
```



```
    13125223.51444201042531001132043521350213
    3245e855,4231343231032343250e1424334e52e213
    $
    154115450}.34031248142403411152251501111301133213
    2531554113.53045108234005314550415431451315220213
```

It is importannt to remember that the randomness of a phenomenon is not itself random and can often be precisely characterised, usually in terms of probability or expected value. For instance quantum mechanics allows a very precise calculation of the half-lives of atoms even though the process of atomic decay is a random one. More simply, though we cannot predict the outcome of a single toss of a fair coin, we can characterise its general behaviour by saying that if a large number of tosses are made, roughly half of them will show up "Heads". Ohm's law and the kinetic theory of gases are precise characterisations of macroscopic phenomena which are random on the microscopic level.

## Chaos and Chaotic Systems

- Mathematical chaos is a study of simple systems that produce complicated behaviour.
- This is perfect for making music on modular synthesisers, since we all know interesting music is complicated, but we can't implement very complicated equations with the limited resources of modular synths.



## Chaos and Chaotic Systems

The patterns produced by chaotic systems bear some similarities to random noise, such as we use to make snare drums and so forth, but tend to have much more structure": The chaotic patterns tend to be "quasiperiodic" that is, the output of the system follows a regular patterni, but not exactly.

From cycle to cycle, the patterns are slightly different. Obviously, totally random noise is not too useful for specifying melodies and thythes (although some may argue to the contrary). But chaotic patterns seem to have just the right mix of randomness and repeatability to produce interesting music.


## Chaos and Chaotic Systems

The first patch we will look at implements four simple non-linear ecosystems, defined by the formula that Mitchell Feigenbaum used to come to his 'Chaos'-theory. The equation that governs this chaotic system is

$$
X(t+1)=G * X(t){ }^{*}(1-X(t))
$$

This equation is known as the Logistic Equation. It was first used to model the population of animal species from year to year. In the equation above $X(t)$ is the ratio of the actual population to the maximum population. Each iteration (e.g. from one year to the next) gives the new relative population in terms of the old one.

## Chaos and Chaotic Systems

$$
X(t+1)=G * X(t) *(1-X(t))
$$

The parameter $G$ is the effective growth rate. The two terms in the equations model the fact that more animals will have more offspring (so growth is proportional to $X$ ), but will compete for resources (so that growth is also proportional to $(1-X)$ ). As the population increases the load on the environment increases reducing the availability of resources and limiting the growth rate.

## Chaos and Chaotic Systems

This is modeled by the $(1-X(t))$ term. For growth rates $G$ less than $1, X(t)$ tends to 0 . For growth rates between 1 and 3, $X(t)$ tends to $1-1 / G$. Beyond $G=3$ a bifurcation occurs causing an oscillation from year to year. (corresponding to high and low populations in alternate years). Further bifurcations occur until at $G=3.53 \ldots$ chaotic dynamics set in. For values of $G$ greater than 4 the system diverges (goes off to -infinity, or saturates at the most negative possible value).

## Chaos and Chaotic Systems

What does all this animal husbandry have to do with the dog-eatdog world of musical synthesis? Well, the chaotic behaviour exhibited by the logistic function can be used to produce interesting music. In the patch below the $X$ values are used to play notes, but they could be used to control anything. In the patch there are four separate chaotic systems implemented. The systems can influence each other. They can be helped by their right neighbours to become more alive while their growth can be suppressed by their left neighbours. Sounds like real life, don't it? Well, actually it sounds like the Nord Modular, so don't worry, no need for peace keeping forces in your machine. Some settings can make all four systems slowly succumb, others can result in chaotic behaviour, and somewhere in between there should be "easy livin'". In figure 10.1 we show the behaviour of the logistic equation for various settings of the G parameter.

## Choice and Chance

## Chaos and Chaotic Systems

The behaviour of the logistic equation for various settings of the G parameter. For G less than 3, the system quickly settles to a fixed value. For G between 3 and 3.53 oscillations occur. For G greater than 3.53 chaotic dynamics result.

## Choice and Chance

## Chaos and Chaotic Systems

For the equation to work it can not start with a zero value for X , as nothing can grow out of nothing unless some "Act of God" is involved. So a provision is made that a small value is automatically added to $X$, only in the first cycle when $X$ is zero due to patch loading conditions.

## Chaos and Chaotic Systems

The following patch implements another type of Chaotic system, this one termed the Henon strange attractor. This system was discovered by the French astronomer Michel Henon while studying the dynamics of stars moving within galaxies. The equations for this system are:
$x(n+1)=1+y(n)-a^{*} x(n)^{*} x(n)$
$y(n+1)=b * x(n)$
The Henon system is 2-dimensional, unlike the logistic system described earlier, which was 1-dimensional. That means it produces two values which you can use to control various aspects of the music. As with the logistic equation, certain settings of the parameters $a$ and $b$ give rise to chaotic behaviour. An example is shown in figure 10.3, which plots the values of $x$ and $y$ for a setting of $a=1.4, b=0.3$. Notice that the points are distributed along a smooth curve, so that overall pattern of the values of $x$ and $y$ is appears non-random, and repeatable. At any given time, however, the values are essentially randomly distributed along this curve. Thus we have a "structured randomness", just the thing we are looking for in algorithmic music!

Brief overview

Brief overview

BBC Documentary about William
Burroughs: "Nothing is True, Everything is Permitted"

Brion Gysin and William S. Burroughs - Rub Out The World (1965)


Cage

Tudor

Mysticism
Chaos/Cosmos

## Bibliography

http://bibliodyssey.blogspot.com/2006/07/visual-context-of-music.html http://www.blockmuseum.northwestern.edu/picturesofmusic/

Presentations

```
ACMP
Algorithms and Generative Music
Bioacoustics
Chance
CMP
Early Learning
Graphic Abstraction
Installation Art
Interface Design
Judge Proulx's Ruling
Live
Make Your Own
Metaform and Metaforming
Notation as Art
Painting Music
pTech
Sonic Art
Sonic Art Examples
Sonification
sTech
Synaesthesia
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## end

